# The Motions of the Mathematical Mind: On the Instruction of Mathematics to the Mathematically Impaired

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In order for this explanation to make sense, it will require the acceptance of a concept that is unfamiliar to a large number of people. The concept to which I refer is the idea of a physical movement being enjoyable for its own sake. This apparently flighty statement, which seems on the surface to have nothing to do with mathematics, is nonetheless essential to my argument. Its importance will be made clear fairly quickly.

When a mathematician expresses to someone the idea that mathematics is beautiful in and of itself, that mathematician usually receives puzzlement and/or outright scorn for such an idea. Similarly, it is a stretch for some people to believe that a movement as simple as lifting one's arm a few inches can be, independent of any specific task to which it aspires, interesting and worthy of notice. As you would have one of your students take your statement on faith, subject to a later verification, I ask you to do the same with mine. It will be easy enough to verify or disavow the idea at a later date.

Teachers of mathematics have the cards stacked against them early on. Faced with a variety of children each with their own level of ability, these teachers are forced to attempt to instruct these students uniformly in the understanding of abstract ideas. At the beginning, at least, these ideas can be easily represented by concrete examples that children come across every day. How many fritos will they have left if they given seven of them to their friend?

The introduction of algebra divides a certain class of students from another class. It is at this point that the students begin to attempt to conceive of the idea of an unknown. A student must be able to abstract, to work with ideas that have no physical manifestations. Those that readily take to this type of learning do very well. The rest fall short and either struggle with or fall prey to a sense of frustration. At some point, these students believe that they are incapable of learning mathematics. Frustration turns to indifference, which leads to a general dislike, or even mistrust, of mathematics. Another such gap occurs at the level of calculus. And so on, through the ever extending levels of mathematical experience.

In a sense, a similar experience occurs in the physical world, with the use of the body. Some of us learn early on that we cannot throw, or that we are incapable of running and catching a football at the same time. Some of us find that life-drawing is a skill that is lost to us, or that the playing of a musical instrument is somehow just beyond our reach. Our inability to understand why we cannot perform these tasks leads to a resignation about our potential to learn more about them. Convinced that we are separated from "those who can" by an invisible, yet unbreachable barrier, we turn towards those things that we come by easily.

Moshe Feldenkrais was able to develop a method whereby these physical tasks could be learned, and made easy. He determined that if one was willing to take a physical activity apart and examine its pieces in the minutest detail, one could gain a much better understanding of what is really occurring in the body when such activities are taking place. When one moves slowly and gently, in other words, and examines pieces of an activity, the movements which make up that activity, one gains an understanding of the interrelationship of the skeleton and the musculature that one has been previously denied.

The process of learning is better served when one is temporarily indifferent to the task to which one aspires. By examining a movement for its own sake, by allowing one's curiosity about the turning of the wrist to surface, one discovers more about the movement than one would have discovered had one insisted on relating it to the task at hand. One temporarily gains an aesthetic appreciation of ones body. It is through this aesthetic appreciation that one is able to learn most effectively.

The entire process can be nicely described in mathematical terms. Let us start with a function. I use the term "function" in both the literal and the mathematical sense. We are lying on our backs and we want to roll to our sides and sit up. The movement of rolling to our sides can be described as a function of movement through space over time. At any point one can determine where every part of the body is at that time.

Let us say for the moment that we have only a vague idea of that function. We know it is continuous, for there are obviously no breaks in the movement. We know it is contained within a bounded domain and range; we start lying down, and we end sitting up. Let us say it is a real-valued function, as a description of the body at any point may be reduced to a numerical plotting of real numbers on an axis.

We wish to know more about that function. We wish to map it. In some sense, we have a few "pairs" of numbers, that is, we have a vague sense at a number of places on that curve of where we are, and of what our body is doing. But we wish to map it more completely. How can we describe this function at every point?

Moshe Feldenkrais would have us begin to take the function apart, to begin to describe smaller and smaller parts of that function. He would have us examine those parts of the function that are well-behaved and those that are not. In short, he would have us differentiate the movements in our bodies.

It is no accident that such a word has been chosen. Moshe Feldenkrais was a nuclear physicist and was fully versed in higher mathematics. He chose to name the understanding of our functionality as Functional Integration. And differentiation is as much a part of the understanding of movement as it is the understanding of a function.

Sadly, we are dealing with a real-world situation, and our ability to differentiate only goes so far. We can make the sections of our function that we wish to examine smaller and smaller, but we cannot truly make them infinitesimal. In a sense, our nervous system must take the limit for us, and assist us in doing what we cannot consciously do ourselves. But we instruct the nervous system to take that limit by indulging our curiosity about smaller and smaller pieces of our function.

When we have spent some time examining the pieces of this function that we would most like to explore, it behooves us to put the pieces back together, having, as we do, a more complete picture of our function. So we take the innumerable pieces of our function and we integrate them. We describe the domain over which our bodies travel as the function we undertake to complete. We roll over and sit up now with a much better picture of that function. Subsequently, we roll over and sit up with much greater ease, using connections in our bodies that we had discounted previously, or that we had been unaware of.

In the process of learning about our function, we have improved it. We did not have to direct this improvement; the process itself sufficed to improve our function. If we had approached that function with our own ideas of what it must look like, we would have inevitably kept ourselves from determining its true appearance. If our students of mathematics insist that, at some place on the curve $f(x)=X^2$ there must lie a pair (3,6), we know that, at least at that point, they will fail to recognize the true nature of that function.

I cannot demonstrate to any reader of this article immutable proof that the process which I have just described works. I can only say that this is a method which has been developed by a man of science over a period of fifty years and which has been subsequently explored and put to good use by a number of professionals in the United States and the world. I have my own positive experiences to recommend themselves, and I am convinced that, for the improvement of physical function, this process works. I leave it to the individual readers to test the validity of Feldenkrais's ideas for themselves.

But if one has gone this far with me, and will accept, at least for the sake of argument, that in some cases this method has provided for some people a method whereby they were able to learn what they perhaps assumed was out of their reach, then one will perhaps go the rest of the distance with me and explore the further possibilities of this idea.

When one is faced with the task of teaching someone a physical activity, one has recourse to the movements within that physical activity for instruction. One can direct a student's attention to parts of their body. One can see those body parts moving. One can (given permission, of course) touch those parts of the body and move them to better describe a function to a person.

But what can be done with the task of teaching someone an abstract idea? How can we look at someone's mind, a young child's mind, perhaps, and say, "Oh, no, you're missing something. When you think about delta-x, this part of your mind needs to move over here." If we could do such a thing, would we improve our ability to teach mathematics?

Let me phrase it another way. If we were more aware of what the process of our mathematical learning was, could we more effectively teach that process to others? I believe the answer is "Yes." Furthermore, I believe that the same method which can be used to elucidate a function of the body can make clear the processes which occur seemingly in the brain alone.

What is the process by which we learn mathematics? If I could answer that question definitively, I would win a Nobel prize. Alas, I cannot answer that question, but I can do the next best thing. I can describe a way of exploring how each one of us, individually, learns mathematics.

We begin with a function. Let us say that function is, "I wish to understand that 2+2=4." The function is the acquisition over time of the understanding that we can add two integers together and get a third, and that that third integer will always be the same.

As children, nearly all of us, even those with severe mental disabilities, are able to come to this understanding. It is quite a remarkable thing, too, for the gap between knowing no mathematics at all and the understanding of the relation of a number of abstract symbols which describe a real world situation is an infinite one which somehow we are able to bridge. We cannot, as human beings, fully describe every piece of that function. But we can take parts of it and examine them.

A child comes across numerous examples of 2 throughout its early life. A number of people explain to the child that it has 2 eyes, 2 ears, 2 hands. There are 2 headlights on the car. When you march, you step "one-two." We do not know where or how the connection takes place, but at some point, that child realizes that each type of 2 is in fact the same type of two, and that this number, "2," describes an unchanging quantity. That quantity can be counted, and it does not matter in which direction one counts.

At another point in the child's life, it discovers that there are other quantities that are represented by other numbers, and that once one assigns a symbol to these quantities, that symbol does not change. One has 5 fingers. There are 10 commandments. I have 11 brothers and sisters. Once again, the child discovers that it does not matter which brother or sister is counted first, the sum of those countings will always be the same (barring further activity from the parents).

Finally a day comes when the child is shown that the symbol "4," which always represents four of something, can be described another way. Four can be described as "2+2." The child has had the experience of counting to 2, and so knows what 2 is. Also, the child has counted to 4, and knows what four is. But now the child discovers that, while counting to 4, he or she can stop at the number 2, and then continue counting the rest of the items in that set, starting over at 1. In other words, "One block, two blocks. Stop. Another one block, another two blocks." The child knows there are four blocks. The child has also seen that there are two countings of two blocks. Suddenly, the child realizes that there is a symbol to describe the putting together of these two sets of two blocks, and that this symbol always means the same thing.

The development of skills in addition continue with the understanding of the equals-sign, and then with the understanding that this game of combining countings can be played with more than just two and two.

I am not suggesting that every child learns in this way, or in this order. I am suggesting, rather, that this may be the process by which one child has learned to add two numbers. And that, when the child is older, the child will use a similar process in tackling bigger issues, not limited to mathematics.

If the child is a sufficiently curious adult, or young adult, that child may go back and examine, to the best of its ability, exactly how it figured out what it now knows. The process will be limited by the person's understanding of his or her own learning process, and by the person's ability to tolerate their own self-image at that moment.[1]

[1]The intolerance of one's self-image is another subject entirely, but it must be recognized briefly as a barrier to

But in the examination of that process, of the steps towards understanding, one will gain a better picture of how one learns what one learns.

Furthermore, as one's understanding of one's own process deepens, one can break each step into smaller and smaller parts, always remembering for the sake of re-integration what the actual function is that one is examining. As in the case of a physical activity it is useful and very helpful to temporarily put the need to grasp the concept out of one's head and dwell for a while in the realm of pure curiosity, so, I believe, it will be in the case of teaching a mathematical skill.

It may be difficult for some to see the point in breaking apart the steps for the acquisition of the understanding of "2+2=4." Allow me to relate the process to a higher level of understanding, so that one can see its use in the teaching of more instructable mathematics, mathematics which almost always requires the assistance of a teacher. I am speaking of the Calculus.

When one is teaching the calculus, one is nearly always teaching it to people who are old enough to have a conscious sense of their ability to learn. Whether or not they deem themselves good learners, they are likely to have some unspoken opinion about how well they can learn a particular subject, including Physical Education, and including Mathematics.

These students, be they high-school or college students, usually do not see Calculus as a confluence of several important but elusive concepts such as taking a limit, continuity, and the definition of a "real number." Instead, they see it as something

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curiosity. If one cannot tolerate one's slow mathematical learning ability, then one will never have the desire to explore what one is capable of learning at a given time, as it will remind oneself of this perceived defect. This self-intolerance begins in mathematics classes every day and is carried along by most people for the rest of their lives.

whole and indivisible, which one gets or doesn't get, and, subsequently, which one passes or does not pass. Most of them pass it, but still do not "get it," for they tend to forget it very quickly, as can easily be ascertained by questioning, at the end of the summer, any ten people who passed calculus in high-school the previous spring.

Is it possible to teach these students in such a way that they maintain an implicit understanding of the subject they have learned, so that they can always use what they know, in the way that they use "2+2=4?" I believe that the answer is yes, and that the answer lies in the process of the teaching itself.

Let us start with a Function (I use the word in a literal sense here) that we wish to undertake, such as, "I wish to understand the Fundamental Theorem of Calculus." This is a basic but enormously difficult task to undertake, one which usually takes years of higher math study and some retrospection to consummate. We could, of course, use any number of simpler problems such as, "I wish to know how fast I am going after driving three minutes in my car at a steady acceleration." I use the larger problem instead as an example of something akin to a physical activity which we wish to improve upon. One never learns the final way to "throw a ball," one only improves and improves one's ability to throw it. Similarly, one never, once and for all, understands the entire spectrum of ramifications of the Fundamental Theorem, but only deepens ones appreciation for it over time.

Given our Function, we are faced with an inability, or more often, an imperfect ability to fulfill it. Most students can grasp something about this theorem, even if it is only that it *is* the Fundamental Theorem. If we wanted to improve someone's ability to throw, even if that person could only move his or her arm a direction and let the ball go, we could begin by examining some of the elements of the function. When are you letting the ball go? How are you holding the ball? How are your feet placed? Are your eyes open??

The question remains, what are the elements to understanding the Fundamental Theorem? There are hundreds of elements, of course, and for each person, different elements will be the problem. It will become the teacher's task to help the student discover, and consistently identify, which elements interfere with a continuation towards the completion of the Function.

Let us start with some basics that will have occurred to many teachers. "Are you studying with music on? Is that music helping you or hindering you from thinking clearly? Do you do your math homework last or first, and does this help or hinder you? Does it help to walk around when you think?" These basic questions, which can be asked in the classroom, will remind a student that there is a process towards the study of mathematics, and that process includes the state of their bodies.

But we can go further than this. We can begin to ask questions about the state of a person's mind as they study. Let us pretend that we have taken a seat to read the Fundamental Theorem with an eye towards understanding it. Let us pretend that we have done what we can to make this task physically easier for ourselves. We begin with the first sentence.

"If f(x) is continuous on a closed interval [a,b]..."

I would stop right there. Those of a mathematical bent will probably pound their heads in frustration, as we have failed even to get to the end of the first statement. But I would beg the indulgence of the mathematically competent; for I maintain that most students will be quite lost by the time they reach the closing parentheses after "b." Now, despite their confusion, the students will continue to read on, taking for granted what they have already read as trivial. And many will be able to make some sense out of the complete sentence, and furthermore the entire proof. But a great many of them, capable of reading the proof and understanding each and every symbol in it, will be incapable of solving a word problem on the very next page.

In this case, we have a completed Function: "I have read and understood, a little, the Fundamental Theorem of Calculus." But it has been completed poorly. And if we are careful in our observations of our own mind, we can begin to discover that we do not know what f(x) really means, that our personal definition of a function is a little cloudy. Perhaps we understand the word "continuous" as a metaphor, but not as a mathematical concept.

These observations are vital. They are the movements which can be improved. A skilled mathematical mind will gloss over the opening sentence with a good sense of what f(x) is, and what "continuous" versus "discontinuous" means, and that a function which is bounded is very different from one that is not. The skilled mathematical mind is as adept at going through these mental motions as a good athlete is at throwing a ball towards home plate; so adept, in fact, that they may no longer notice the movements taking place. When such motions are easy, there is no need to examine them. But when they are difficult, as they may be for the English Major who writes beautiful papers on Shelley but misses seven out of every twelve questions on his math quizzes, then it is of enormous value to slow down, and to examine each motion for its defects.

We have, in this first section of this first sentence, three elements: the definition of a function, continuity, and the idea of a closed interval. Each concept can be differentiated from the whole, and explored. The level of exploration must vary from person to person, depending on their experiences and their sense of curiosity. It is helpful, when teaching physical activities, to insist that the movements be done slowly, and always on such a small level that they are easy. In this case, perhaps we might start with the simple motion of understanding that if a line moves one notch up for every notch it moves to the right, then we can describe the line easily; that that description will function in a way that provides us, consistently, a number for moving left-and-right, and a number moving up-and-down, which will always be the same. Perhaps the student will grasp this readily, and one can move to a subject only a little bit more difficult, proceeding onwards until some fogginess is reached. Or perhaps the student is already fogged, and one must move back a little bit until one reaches a noticeable change in that student's ability to comprehend.

When one has explored a concept for a little while, one can return to the larger picture, try the overall task again, and note the level of improvement. This process of exploration can be done on any level, and with any element. The limits of this exploration are the patience of the student and the teacher, and the constraints of the teaching situation.

If one has an opportunity to consistently teach a student to notice the sticking points in the understanding of a mathematical idea, then the student can begin him or herself to notice those sticking points. Suddenly, the student is engaged in understanding where they are having trouble, and the teacher has a partner in instruction.

I would expect that most teachers of mathematics would be somewhat impatient with this approach. They might complain that it is far too easy, given such a complex task as learning Calculus, to get sidetracked in all the details and lose sight of the final goal. Furthermore, they might observe that the students themselves do not want this kind of training. Most of them wish to know merely what they must know to pass the class, and it becomes a teacher's job to teach them something in spite of this attitude. Finally, the realities of the classroom situation make such an approach impractical or even impossible, as individual attention would be required for every student and also an unidentifiable amount of time to learn the subject.

I do not deny the validity of these arguments. Clearly we are not, today, set up to teach mathematics in such a way. But if one can admit that such an approach might be an effective way of teaching mathematics, given the proper environment, then we can perhaps explore the degree to which this approach might have merit. Perhaps the ideas in this paper, implemented by patient and curious teachers, might serve to do what established methods of teaching have failed to do for a majority of mathematics students, that is, help them through the difficult aspects of their learning.

Of course, there is no reason why those with a measure of mathematical aptitude cannot try this approach with a willing student. The approach could be explored over a school-year's time in bi-weekly segments of 45-minutes each. The student and the teacher would have nothing to lose by the exploration of this method of teaching, both agreeing that the desire to improve is the only goal, and that any progress at all, over any period of time, is worth having.

If such an experiment is carried out, I would caution the teachers in two things. First, it is necessary that frequent rests be offered to the student, within each teaching period. By frequent, I mean, as often as a student wants to rest. Resting is as important to further learning as breathing is to filling a balloon with air. The student will learn best that learns gradually, without too much strain. Second, it is essential that a teacher teaching in this way remain neutral towards his or her student. By this, I mean that the teacher should have a simple curiosity about what is possible with this student, and should encourage that curiosity in the student as well. Judgment about whether or not something *should* be learned at "this point in time" will only hamper the student's discovery of what is actually happening at a given moment.

Adam Cole publishes a free monthly newsletter about *Feldenkrais* on www.feldenkraisinfo.com. During his training, Adam wrote a novel entitled *The Myth of Magic*, about a school of magicians who are fighting for their survival in a world that does not understand them. Visit www.mythofmagic.com to read an excerpt from this Feldenkrais-influenced book. To learn more about Adam, hear his CD, read his poetry, and much more, visit www.acole.net.

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